Class X Session 2025-26
Subject - Mathematics (Standard)
Sample Question Paper - 04

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Two coins are tossed simultaneously. The probability of getting at most one tail is: **[1]**

   a) $\frac{3}{4}$ b) 1

   c) $\frac{1}{4}$ d) $\frac{1}{2}$

2. The product of two consecutive even integers is 528. The quadratic equation corresponding to the above statement, is **[1]**

   a) (1 + x) 2x = 528 b) 2x (x + 4) = 528

   c) x (x + 2) = 528 d) 2x (2x + 1) = 528

3. The volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is **[1]**

   a) 77.6 cm^3 b) 58.2 cm^3

   c) 9.7 cm^3 d) 19.4 cm^3

4. If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then the value of k is ________. **[1]**

   a) 2 or -2 b) -2 or 0

   c) 0 only d) 2 or 0

5. If 18th and 11th terms of an A.P. are in the ratio 3 : 2, then its 21st and 5th terms are in the ratio **[1]**

   a) 3 : 1 b) 3 : 2

   c) 1 : 3 d) 2 : 3

6. The abscissa of any point on the x-axis is **[1]**

a) 0

b) 1

c) x

d) – 1

7. If the sum of the zeroes of the quadratic polynomial $kx^2 + 2x + 3k$ is equal to their product, then k equals. [1]

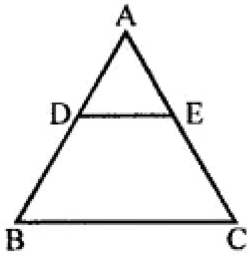
a) $\frac{2}{3}$

b) $-\frac{2}{3}$

c) $-\frac{1}{3}$

d) $\frac{1}{3}$

8. In the given figure, DE||BC. If DE = 5 cm, BC = 8 cm and AD = 3.5 cm then AB = ? [1]



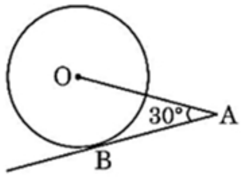
a) 4.8 cm

b) 5.6 cm

c) 6.4 cm

d) 5.2 cm

9. In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and $\angle OAB = 30^o$, then the radius of the circle is: [1]



a) $\sqrt{3}$ cm

b) 2 cm

c) $3\sqrt{3}$ cm

d) 3 cm

10. If four sides of a quadrilateral ABCD are tangential to a circle, then [1]

a) AC + AD = BC + DB

b) AC +AD = BD + CD

c) AB + CD = BC + AD

d) AB + CD = AC + BC

11. $\frac{\sin\theta}{1+\cos\theta}$ is equal to [1]

a) $\frac{1-\cos\theta}{\sin\theta}$

b) $\frac{1-\sin\theta}{\cos\theta}$

c) $\frac{1-\cos\theta}{\cos\theta}$

d) $\frac{1+\cos\theta}{\sin\theta}$

12. The LCM of smallest 2-digit number and smallest composite number is [1]

a) 20

b) 4

c) 12

d) 40

13. The angle of depressions of the top and bottom of 10 m tall building from the top of a multistoried building are 30^o and 60^o respectively. Find the height of the multistoried building and the distance between the two buildings. [1]

a) 16 m, $5\sqrt{3}$ m

b) 16 m, $4\sqrt{3}$ m

c) 15 m, $6\sqrt{3}$ m

d) 15 m, $5\sqrt{3}$ m

14. The hour hand of a clock is 6 cm long. The area swept by it between 11.20 am and 11.55 am is [1]

a) 11 cm^2

b) 2.75 cm^2

c) 10 cm^2 d) 5.5 cm^2

15. The length of a minute hand of a wall clock is 7 cm. What is the area swept by it in 30 minutes is **[1]**

a) 63 cm^2 b) 35 cm^2

c) 77 cm^2 d) 50 cm^2

16. A month is selected at random in a year. The probability that it is March or October, is **[1]**

a) $\frac{1}{4}$ b) $\frac{1}{12}$

c) $\frac{3}{4}$ d) $\frac{1}{6}$

17. In a single throw of a pair of dice, the probability of getting the sum a perfect square is **[1]**

a) $\frac{1}{18}$ b) $\frac{1}{6}$

c) $\frac{7}{36}$ d) $\frac{2}{9}$

18. Using empirical relationship, the mode of a distribution whose mean is 7.2 and the median 7.1, is: **[1]**

a) 6.9 b) 6.3

c) 6.2 d) 6.5

19. **Assertion (A):** If we join two hemispheres of same radius along their bases, then we get a sphere. **[1]**

Reason (R): A tank is made of the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and radius is 30 cm. The total surface area of the tank is 3.3 m^2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** a, b, c are in A.P. if and only if 2b = a + c. **[1]**

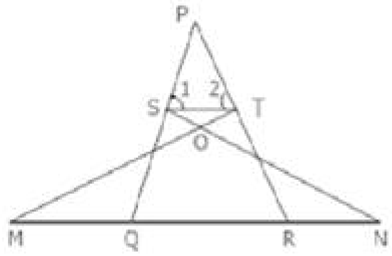
Reason (R): The sum of first n odd natural numbers is n^2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

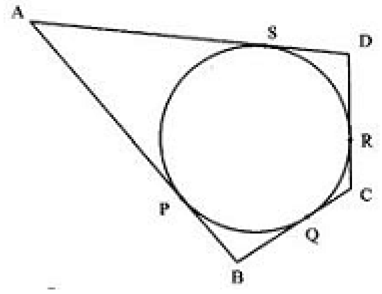
Section B

21. Prove that $6 + 3\sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number. **[2]**

22. In the given figure, if $\angle 1 = \angle 2$ and $\triangle NSQ \cong \triangle MTR$ .Then prove that $\triangle PTS \cong \triangle PRQ$ **[2]**



23. A quadrilateral ABCD is drawn to the circumference of a circle. Prove that: AB + CD = AD + BC **[2]**



24. Prove the trigonometric identity: $\frac{\cos\theta}{(1-\tan\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)} = (\cos\theta+\sin\theta)$ **[2]**

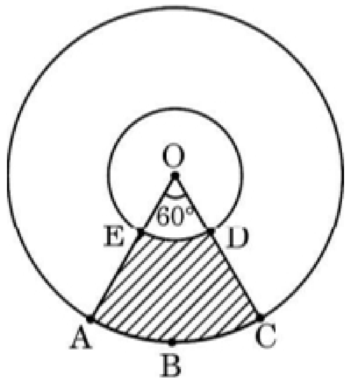
OR

If 3 cot A = 4, find the value of $\frac{cosec^2 A+1}{cosec^2 A-1}$.

25. Three horses are tied each with 7 m long rope at three corners of a triangular field having sides 20 m, 34 m and 42 m. Find the area of the plot which can be grazed by the horses. **[2]**

OR

In the given figure, two concentric circles with centre O are shown. Radii of the circles are 2 cm and 5 cm respectively. Find the area of the shaded region.



Section C

26. A shopkeeper has 120 litres of petrol, 180 litres of diesel and 240 litres of kerosene. He wants to sell oil by filling the three kinds of oils in tins of equal capacity. What should be the greatest capacity of such a tin? **[3]**

27. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, -3) and (3, 4). Find its centroid. **[3]**

28. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides. **[3]**

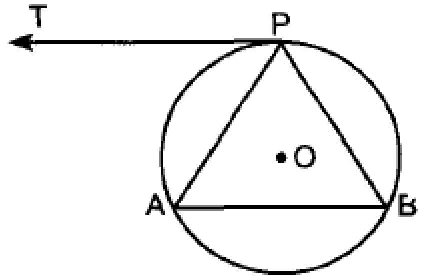
OR

The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

29. Prove that parallelogram circumscribing a circle is a rhombus. **[3]**

OR

A tangent PT is drawn parallel to a chord AB as shown in figure. Prove that APB is an isosceles triangle.



30. If $\angle B$ and $\angle Q$ are acute angles such that sin B = sin Q, then prove that $\angle B = \angle Q$. **[3]**

31. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive mile **[3]**

stones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$.

Section D

32. 36 pens and 24 pencils together cost ₹ 780, while 24 pens and 36 pencils together cost ₹ 720. Find the cost of one pen and of one pencil. **[5]**

OR

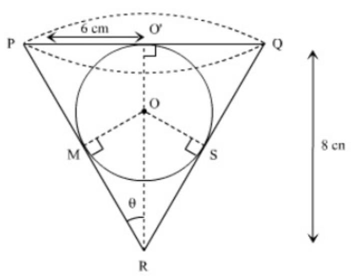
On selling a T.V. at 5% gain and a fridge at 10% gain. A shopkeeper gains Rs. 2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss. He gains Rs. 1500 on the transaction. Find the actual price of the T.V. and the fridge.

33. **[5]**
   i. Derive section formula.
   ii. In what ratio does the point (-4, 6) divide the line segment joining the points A(-6, 10) and B(3, -8).

34. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. **[5]**

OR

A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows?

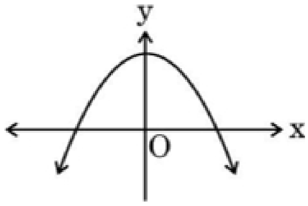


35. The sum of first n terms of an A.P. is $5n^2 + 3n$. If its m^{th} term is 168, find the value of m. Also, find the 20^{th} term of this A.P. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Rainbow is an arch of colours that is visible in the sky after rain or when water droplets are present in the atmosphere. The colours of the rainbow are generally, red, orange, yellow, green, blue, indigo and violet. Each colour of the rainbow makes a parabola. We know that any quadratic polynomial $p(x) = ax^2 + bx + c$ $(a \neq 0)$ represents a parabola on the graph paper.



i. The graph of a rainbow y = f(x) is shown in the figure. Write the number of zeroes of the curve. (1)

ii. If the graph of a rainbow does not intersect the x-axis but intersects y-axis at one point, then how many zeroes will it have? (1)

iii. If a rainbow is represented by the quadratic polynomial $p(x) = x^2 + (a + 1) x + b$, whose zeroes are 2 and -3, find the value of a and b. (2)

OR

The polynomial $x^2 - 2x - (7p + 3)$ represents a rainbow. If -4 is a zero of it, find the value of p. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Mutual Fund: A mutual fund is a type of investment vehicle that pools money from multiple investors to invest in securities like stocks, bonds or other securities. Mutual funds are operated by professional money managers, who allocate the fund's assets and attempt to produce capital gains or income for the fund's investors.

Net Asset Value (NAV) represents a fund's per share market value. It is the price at which the investors buy fund shares from a fund company and sell them to a fund company.

The following table shows the Net Asset Value (NAV) per unit of mutual fund of ICICI mutual funds:

NAV (in ₹)	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Number of mutual funds	13	16	22	18	11

Based on the above information, answer the following questions:

i. What is the upper limit of modal class of the data?

ii. What is the median class of the data?

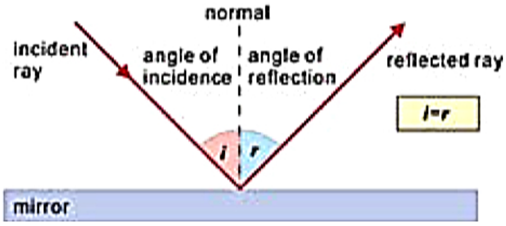
iii. a. What is the mode NAV of mutual funds?

OR

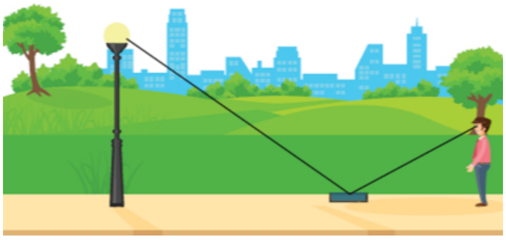
b. What is the median NAV of mutual funds?

38. **Read the following text carefully and answer the questions that follow:** **[4]**

The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Suresh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Suresh's eye level is 1.5 m above the ground. The distance of Suresh and the pole from the mirror are 1.8 m and 6 m respectively.



i. Which criterion of similarity is applicable to similar triangles? (1)
ii. What is the height of the pole? (1)
iii. If angle of incidence is i, find tan i. (2)

OR

Now Suresh move behind such that distance between pole and Suresh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Suresh? (2)

Solution

Section A

1. **(a)** $\frac{3}{4}$

 Explanation:

 At most one Tail

 Favourable case $\rightarrow$ HH, HT, TH = 3

 p(at most one Tail) = $\frac{3}{4}$

2. **(c)** x (x + 2) = 528

 Explanation:

 Let the first number = x

 Second number = x + 2

 According to question

 x(x + 2) = 528

3. **(d)** 19.4 cm^3

 Explanation:

 R = $\frac{4.2}{2}$

 = 2.1cm

 h = 4.2 cm

 Volume of cone= $\frac{1}{3}\pi r^2 h$

 $\frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4.2$

 = 19.404 cm^3

4. **(a)** 2 or -2

 Explanation:

 Since the roots are equal, we have D = 0.

 $\therefore 36k^2 - 4 \times 9 \times 4 = 0 \Rightarrow 36k^2 = 144 \Rightarrow k^2 = 4 \Rightarrow k = 2$ or -2.

5. **(a)** 3 : 1

 Explanation:

 Here, 18th term : 11th term = 3: 2

 $\Rightarrow \frac{a_{18}}{a_{11}} = \frac{3}{2} \Rightarrow \frac{a+17d}{a+10d} = \frac{3}{2}$

 $\Rightarrow 2a + 34d = 3a + 30d$

 $\Rightarrow 34d - 30d = 3a - 2a \Rightarrow a = 4d$

 Now $\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$

 $= \frac{24d}{8d} = \frac{3}{1}$

 $a_{21} : a_5 = 3 : 1$

6. **(c)** x

 Explanation:

 Since coordinates of any point on the x-axis is (x, 0)

 Therefore, abscissa is x.

7. **(b)** $-\frac{2}{3}$

Explanation:

$\alpha+\beta=\alpha\beta \Rightarrow \frac{-2}{k}=\frac{3k}{k} \Rightarrow \frac{-2}{k}=3 \Rightarrow k=\frac{-2}{3}$

8.

(b) 5.6 cm

Explanation:

$\because$ DE ∥ BC

$\therefore \frac{AD}{AB}=\frac{AE}{AC}=\frac{DE}{BC}$ (Thales' theorem)

$\Rightarrow \frac{3.5}{AB}=\frac{5}{8}$

$\Rightarrow AB=\frac{3.5\times8}{5}$ =5.6 cm

9.

(d) 3 cm

Explanation:

sin 30^o = $\frac{OB}{OA}$

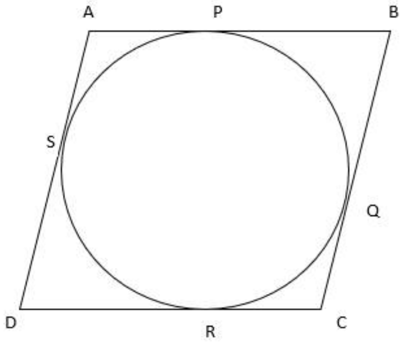
$\frac{1}{2}$ = $\frac{r}{6}$

r = 3 cm

10.

(c) AB + CD = BC + AD

Explanation:



Property: If two tangents are drawn to a circle from one external point, then their tangent segments (lines joining the external point and the points of tangency on circle) are equal.

By the above property,

AP = AS (tangent from A)

BP = BQ (tangent from B)

CR = CQ (tangent from C)

DR = DS (tangent from D)

Now we add above 4 equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ AB + CD = BC + AD [$\because$ AP + BP = AB, CR + DR = CD, AS + DS = AD, BQ + CQ = BC]

Hence, the right option is AB + CD = BC + AD

11. **(a)** $\frac{1-\cos\theta}{\sin\theta}$

Explanation:

We have, $\frac{\sin\theta}{1+\cos\theta}=\frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$=\frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}=\frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$

$=\frac{1-\cos\theta}{\sin\theta}$

12. **(a)** 20

Explanation:

As we know, the smallest two-digit number is 10 and the smallest composite number is 4.

By prime factorisation, we get;

$4 = 2 \times 2$

$10 = 2 \times 5$

Now, LCM of 4, 10 = $2 \times 2 \times 5 = 20$

Therefore, the LCM of the smalles two-digit number and the smallest composite number is 20.

13.

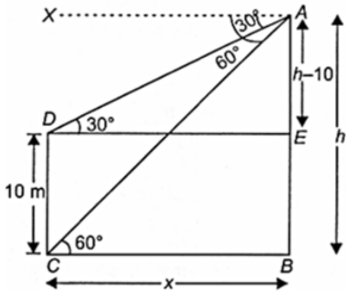
(d) 15 m, $5\sqrt{3}$ m

Explanation:

Let AB be the multi storied building of height h and let the distance between two building be x meters.

In $\triangle$ADE, tan $30^o = \frac{AE}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-10}{x}$ [$\because$ CB = DE = x]

$\Rightarrow$ x = $\sqrt{3}$ (h - 10) ...(i)



In $\triangle$ACB,

$\tan 60^o = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we have

$\sqrt{3}(h - 10) = \frac{h}{\sqrt{3}} \Rightarrow h = 15$ m

From (ii), $x = \frac{h}{\sqrt{3}}$ So, $x = \frac{15}{\sqrt{3}} = 5\sqrt{3}$ m

Hence, height of multi storied building = 15 meters

Distance between two buildings = $5\sqrt{3}$ m

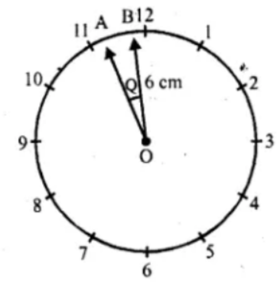
14.

(d) 5.5 cm^2

Explanation:

Length of hour hand of a clock (r) = 6 cm

Time 11.20 am to 11.55 am = 35 minute = $\frac{35}{60}$h



$\therefore$ In 1 hour the hour hand rotates 30°.

Thus, central angle of the sector = $30 \times \frac{35}{60} = 17.5^\circ$

$\therefore$ Area of the sector swept by the hour hand = $\frac{17.5}{360} \times \frac{22}{7} \times 6 \times 6$ cm^2

$= \frac{2.5 \times 22}{10} \text{cm}^2 = 5.5\text{cm}^2$

15.

(c) 77 cm^2

Explanation:

For a minute hand, 60 minutes is equivalent to 360^o and so 30 minutes will be 180^o.

Area swept in 60 minutes is area of full circle.

So area swept in 30 minutes will be area of half circle.

Thus, area swept $= \frac{1}{2} \times \left(\frac{22}{7}\right) \times 7^2$ = 77 cm^2

16.

(d) $\frac{1}{6}$

Explanation:

No. of months in a year = 12

Probability of being March or October = $\frac{2}{12}$ = $\frac{1}{6}$

17.

(c) $\frac{7}{36}$

Explanation:

A pair of dice is thrown simultaneously

$\therefore$ No. of total events (n) = 6 $\times$ 6 = 36

Total outcomes ,

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ Event whose sum is a perfect square are (1, 3), (2, 2), (3, 1), (3, 6), (4, 5), (6, 4), (6, 3)

$\therefore$ m = 7

$\therefore$ Probability = $\frac{m}{n} = \frac{7}{36}$

18. **(a)** 6.9

Explanation:

6.9

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

Section B

21. Let us assume that $6+3\sqrt{2}$ is a rational number

$\therefore 6+3\sqrt{2}=x$ say

$\Rightarrow \sqrt{2}=\frac{x-6}{3}$

Now $\frac{x-6}{3}$ is a rational number

$\Rightarrow \sqrt{2}$ is a rational number

But this contradicts the given fact that $\sqrt{2}$ is an irrational number

$\therefore$ our assumption is wrong

$\Rightarrow 6+3\sqrt{2}$ is an irrational number.

22. Since $\triangle$ NSQ $\cong$ $\triangle$MTR

$\therefore \angle$ SQN = $\angle$ TRM

$\Rightarrow \angle$ Q= $\angle$ R(in $\triangle$PQR)

$\angle Q$ = 90^o - $\frac{1}{2}\angle P$

Again 1 = ∠ 2 [given in △ PST](Isosceles property)

∴ ∠ 1= ∠ 2 = $\frac{1}{2}(180^\circ - \angle P)$

$= 90^\circ - \frac{1}{2}\angle P$

Thus, in △PTS and △PRQ

∠ 1 = ∠ Q [Each = 90° - $\frac{1}{2}\angle P$]

∠ 2 = ∠ R, ∠ P= ∠ P(Common)

△ PTS ≅ △PRQ

23. Let the sides of the quadrilateral ABCD touch the circle at P, Q, R and S. Since, the lengths of the tangents from an external point to a given circle are equal.

∴ AP = AS

⇒ BP = BQ

CR = CQ

⇒ DR = DS

Adding, (AP + BP) + (CR + DR) = (BQ + CQ) + (AS + DS)

⇒ AB + CD = BC + AD.

Hence proved

24. LHS = $\frac{\cos\theta}{(1-\tan\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$= \frac{\cos\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)}$

$= \frac{\cos^2\theta}{(\cos\theta-\sin\theta)} - \frac{\sin^2\theta}{(\cos\theta-\sin\theta)} = \frac{\cos^2\theta-\sin^2\theta}{(\cos\theta-\sin\theta)}$

$= \frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)} = (\cos\theta + \sin\theta)$ = RHS

Hence proved.

OR

Given,

3 cot A = 4

⇒ cot A = $\frac{4}{3}$

We know that,

$\operatorname{cosec}^2 A - \cot^2 A = 1$

$\operatorname{cosec}^2 A - \left(\frac{4}{3}\right)^2 = 1$

$\operatorname{cosec}^2 A = 1 + \frac{16}{9} = \frac{9+16}{9} = \frac{25}{9}$

Thus,

$\frac{cosec^2 A+1}{cosec^2 A-1} = \frac{25/9+1}{25/9-1} = \frac{34}{16}$

25. Given 3 horses are tethered with 7 m long ropes at three corners of △ABC

Here radius of sectors, r = 7 m

Given sides of △ABC are AB = 20 cm, BC = 30 m, CA = 40 m

Area of the plot which can be grazed = $\frac{x^\circ}{360^\circ} \times \pi r^2 + \frac{y^\circ}{360^\circ} \times \pi r^2 + \frac{z^\circ}{360^\circ} \times \pi r^2$

$= \frac{\pi r^2}{360}[x + y + z]$

$= \frac{\pi r^2}{360} \times 180$ [∵ x + y + z = 180]

$= \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$ = 77 sq. m.

OR

Area of sector OABC = $\frac{\pi\times5^2\times60^\circ}{360^\circ} = \frac{25\pi}{6}$ cm^2

Area of sector OED = $\frac{\pi\times2^2\times60^\circ}{360^\circ} = \frac{4\pi}{6}$ cm^2

Area of shaded region = $\frac{25\pi}{6} - \frac{4\pi}{6} = \frac{21}{6} \times \frac{22}{7}$ = 11 cm^2

Section C

26. The required greatest capacity is the HCF of 120, 180 and 240.

$240 = 180 \times 1 + 60$

$180 = 60 \times 3 + 0$

HCF is 60.

Now HCF of 60, 120

$120 = 60 \times 2 + 0$

∴ HCF of 120, 180 and 240 is 60.

∴ The required capacity is 60 litres.

27. Let P(1, 1), Q(2, -3), R(3, 4) be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of triangle ABC. Then,

P is the midpoint of BC

$\Rightarrow \frac{x_1+x_2}{2} = 1, \frac{y_1+y_2}{2} = 1$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 2$..........(i)

Q is the midpoint of BC

$\Rightarrow \frac{x_2+x_3}{2} = 2, \frac{y_2+y_3}{2} = -3$

$\Rightarrow x_2 + x_3 = 4$ and $y_2 + y_3 = -6$........(ii)

R is the midpoint of AC

$\Rightarrow \frac{x_1+x_3}{2} = 3, \frac{y_1+y_3}{2} = 4$

$\Rightarrow x_1 + x_3 = 6$ and $y_1 + y_3 = 8$..........(iii)

From (i), (ii) and (iii), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$

and, $y_1 + y_2 + y_2 + y_3 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$.........(iv)

The coordinates of the centroid of $\triangle$ABC are

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right)$ [Using (iv)]

28. Let the base of the right triangle be x cm.

Then altitude = (x - 7) cm

Hypotenuse = 13 cm

By Pythagoras theorem

$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$

$\implies x^2 + (x-7)^2 = (13)^2$

$\implies x^2 + x^2 - 14x + 49 = 169$

$\implies 2x^2 - 14x - 120 = 0$

$\implies 2(x^2 - 7x - 60) = 0$ or $x^2 - 7x - 60 = 0$

$\implies x^2 - 12x + 5x - 60 = 0$

$\implies x(x - 12) + 5(x - 12) = 0$

$\implies (x + 5)(x - 12) = 0$

Either x + 5 = 0 or x - 12 = 0

$\implies x = -5, 12$

Since side of the triangle cannot be negative. So, x =12 cm and x = - 5 is rejected.

Hence, length of the other two sides are 12cm, (12 - 7) = 5cm.

OR

Let the denominator be y, then numerators = y - 3

So the fraction be $\frac{y-3}{y}$

By the given condition, new fraction = $\frac{y-3+2}{y+2}$

$= \frac{y-1}{y+2}$

$\frac{y-3}{y} + \frac{y-1}{y+2} = \frac{29}{20}$

$\frac{(y-3)(y+2)+y(y-1)}{y(y+2)} = \frac{29}{20}$

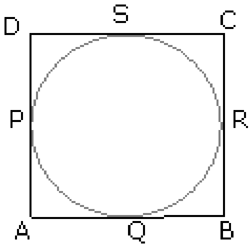
$20[(y-3)(y+2) + y(y-1)] = 29\left(y^2 + 2y\right)$

$20[(y^2 - 3y + 2y - 6) + (y^2 - y)] = 29\left(y^2 + 2y\right)$

$20\left(y^2 - y - 6 + y^2 - y\right) = 29y^2 + 58y$

$20\left(2y^2 - 2y - 6\right) = 29y^2 + 58y$

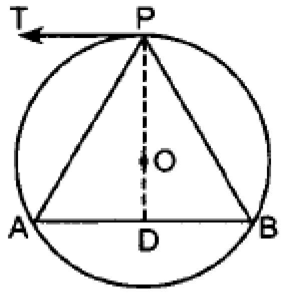
$11y^2 - 98y - 120 = 0$
$11y^2 - 110y + 12y - 120 = 0$
$(11y + 12)(y - 10) = 0$
$\therefore y = 10$
$\therefore$ The fraction is $\frac{7}{10}$

29. 

Given ABCD is a parallelogram in which all the sides touch a given circle
To prove:- ABCD is a rhombus
Proof:-
$\because$ ABCD is a parallelogram
$\therefore$ AB = DC and AD = BC
Again AP, AQ are tangents to the circle from the point A
$\therefore$ AP = AQ
Similarly, BR = BQ
CR = CS
DP = DS
$\therefore$(AP + DP) + (BR + CR) = AQ + DS + BQ + CS = (AQ + BQ) + (CS + DS)
$\Rightarrow$ AD + BC = AB + DC
$\Rightarrow$ BC + BC = AB + AB [$\because$ AB = DC, AD = BC]
$\Rightarrow$ 2BC = 2AB
$\Rightarrow$ BC = AB
Hence, parallelogram ABCD is a rhombus

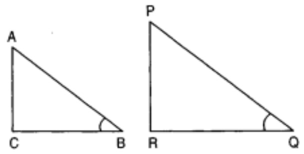
OR

Given,



Construction: Join PO and produce it to D.
Proof: Here, $OP \perp TP$
$\angle OPT = 90^\circ$
Also, $TP || AB$
$\therefore \angle \text{TPD} + \angle \text{ADP} = 180^\circ$
$\Rightarrow \angle \text{ADP} = 90^\circ$
OD bisects AB [Perpendicular from the centre bisects the chord]
In $\triangle$ADP and $\triangle$BDP
AD = BD
$\angle ADP = \angle BDP$ [Each 90°]
$PD = PD$
$\therefore \triangle ADP \cong \triangle BDP$ [SAS]
$\angle PAB = \angle PBA$ [C.P.C.T.]
$\therefore \triangle$PAB is isosceles triangle.

30. Consider two right triangles ABC and PQR in which $\angle B$ and $\angle Q$ are the right angles.
We have,



In $\triangle ABC$

$\sin B = \frac{AC}{AB}$

and, In $\triangle PQR$

$\sin Q = \frac{PR}{PQ}$

$\because \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ...... (i)

$\Rightarrow$ AC = kPR and AB = kPQ .....(ii)

Using Pythagoras theorem in triangles ABC and PQR, we obtain

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$ [using (ii)]

$\Rightarrow \quad \frac{BC}{QR} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(iii)

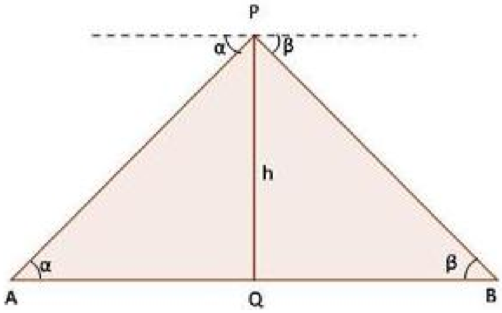
From (i) and (iii), we get

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \Delta ACB - \Delta PRQ$ [By S.A.S similarity]

$\therefore \quad \angle B = \angle Q$

Hence proved.

31. 

Let h be the height of aeroplane above the road and A and B be two consecutive milestone.

In $\Delta\ AQP\ and\ \triangle BQP$,

$\tan\alpha = \frac{h}{AQ}$ and $\tan\beta = \frac{h}{BQ}$

$\Rightarrow AQ = h\ \cot\alpha$ and $BQ = h\ \cot\beta$

$\Rightarrow AQ + BQ = h(\cot\alpha + \cot\beta)$

$AB = h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right)$

As, given that AB = 1 mile

$\Rightarrow h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$

Hence proved.

Section D

32. Let the cost of each pen = ₹ x and the cost of each pencil = ₹ y

$\therefore$ 36x + 24y = 780 or 3x + 2y = 65

and 24x + 36y = 720 or 2x + 3y = 60

Solving and getting x = 15 and y = 10

$\therefore$ cost of each pen = ₹ 15 and the cost of each pencil = ₹ 10

OR

Let the actual price of the T.V. and the fridge be Rs. x and Rs. y respectively.

Then, according to the question,

$\left(\frac{5}{100}x\right)+\left(\frac{10}{100}y\right)=2000$

$\Rightarrow \frac{x}{20}+\frac{y}{10}=2000$

$\Rightarrow$ x + 2y = 40000 ....(1)

And, $\left(\frac{10}{100}x\right)-\left(\frac{5}{100}y\right)=1500$

$\Rightarrow \frac{x}{10}-\frac{y}{20}=1500$

$\Rightarrow$ 2x - y = 30000 .....(2)

Multiplying equation (2) by 2, we get

4x - 2y = 60000 .....(3)

Adding equation (1) from equation (3), we get

5x = 100000

$\Rightarrow x=\frac{100000}{5}=20000$

Substituting x = 20000 in equation (1), we get

20000 + 2y = 40000

$\Rightarrow$ 2y = 40000 - 20000 = 20000

$\Rightarrow y=\frac{20000}{2}=10000$

So, the solution of the given equations is x = 20000 and y = 10000.

Hence, the actual price of the T.V. and fridge are ₹ 20000 and ₹10000 respectively.

Verification. Substituting x = 20000, y = 10000,

We find that both the equation (1) and (2) are satisfied as shown below:

x + 2y = 20000 + 2(10000) = 40000

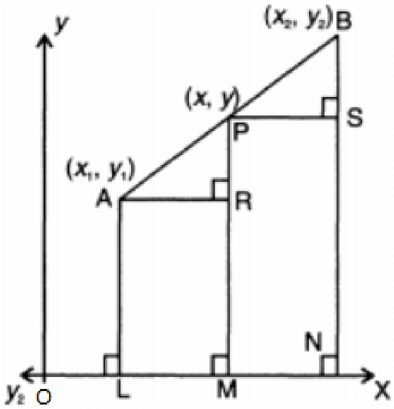
2x - y = 2(20000) - 10000 = 30000

Hence, the solution we have got is correct.

33. i. Derivation of Section Formula:

Let A(x_1,y_1) and B(x_2, y_2) be two points.

Let P(x, y) be a point on line AB, such that P divides it in the ratio $m_1 : m_2$



Let AB be a line segment joining the points A(x_1, y_1), B(x_2, y_2).

Let P have coordinates (x, y).

Draw AL, PM, BN $\perp$ to x-axis.

It is clear from fig., that

AR = LM = (distance between origin and point M) - (distance between origin and point L).

$\therefore$ AR = LM = OM - OL = x - x_1

Similarly, PR = PM - RM = y - y_1

And, PS = (Distance between origin and point N) - (Distance between point M and origin) = ON - OM = x_2- x

Similarly, BS = BN - SN = y_2 - y

$\Delta APR \sim \Delta PBS$ [AAA]

$\frac{AR}{PS}=\frac{PR}{BS}=\frac{AP}{PB}$

Now, $\frac{AR}{PS}=\frac{AP}{PB}$

$\Rightarrow \frac{x-x_1}{x_2-x}=\frac{m_1}{m_2}$

$\Rightarrow m_2(x-x_1)=m_1(x_2-x)$

$\Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$

$\therefore x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$

Similarly, $\frac{PR}{BS} = \frac{AP}{PB}$

$\Rightarrow \frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2}$

$\therefore y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$

$\therefore$ Coordinates of P are $\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

ii. Let (- 4, 6) divides the line segment joining the point. A(-6, 10) and B(3, - 8) in k : 1

So, x_1 = -6, y_1 = 10, x_2 = 3, y_2 = -8, x = -4, y = 6, m_1 = k, m_2 = 1

Using section formula,

$-4 = \frac{k(3) + 1(-6)}{k + 1}$

$\Rightarrow -4k - 3k = -6 + 4$

$\Rightarrow -7k = -2$

$\Rightarrow k = \frac{2}{7}$

Therefore, the ratio = 2 : 7

34. According to the question,a hemispherical depression is cut from one face of the cubical block such that the diameter l of the hemisphere is equal to the edge of the cube.

Let the radius of hemisphere = r

$\therefore r = \frac{l}{2}$

Now, the required surface area = Surface area of cubical block - Area of base of hemisphere + Curved surface area of hemisphere.

$= 6(\text{ side })^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$

$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2}l^2$

$= 6l^2 + \frac{\pi l^2}{4}$

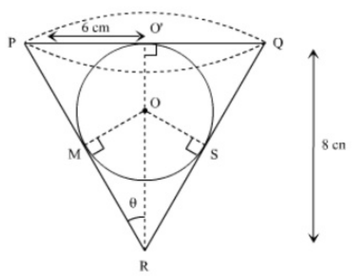
Surface area $= \frac{1}{4}(24 + \pi)l^2$ units.

$= \frac{1}{4}\left(24 + \frac{22}{7}\right)l^2$

OR

Radius (R) of conical vessel = 6 cm

Height (H) of conical vessel = 8 cm



Volume of conical vessel (V_c) = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$

$= 96\pi \text{cm}^3$

Let the radius of the sphere be r cm

In right Δ PO'R by pythagoras theorem We have

$l^2 = 6^2 + 8^2$

$l = \sqrt{36 + 64}$ = 10 cm

In right triangle MRO

$\sin\theta = \frac{OM}{OR}$

$\Rightarrow \frac{3}{5} = \frac{r}{8 - r}$

$\Rightarrow$ 24 - 3r = 5r

$\Rightarrow$ 8r = 24

$\Rightarrow$ r = 3 cm

$\therefore V_1$ = Volume of the sphere ,= $\frac{4}{3}\pi \times 3^3 cm^3 = 36\pi cm^3$

V_2 = Volume of the water = Volume of the cone = $\frac{1}{3}\pi \times 6^2 \times 8cm^3 = 96\pi cm^3$

Clearly, volume of the water that flows out of the cone is same as the volume of the sphere i.e., V_1.

$\therefore$ Fraction of the water that flows out $= V_1 : V_2 = 36\pi : 96\pi = 3 : 8$

35. Here Sum of the n terms of an Arithmetic progression, Sn = $5n^2 + 3n$

$S_1 = 5 \times 1^2 + 3 \times 1 = 8 = t_1$ ....(i)

$S_2 = 5 \times 2^2 + 3 \times 2 = 26 = t_1 + t_2$ ...(ii)

$S_3 = 5 \times 3^2 + 3 \times 3 = 54 = t_1 + t_2 + t_3$ ...(iii)

From (i), (ii) and (iii),

$t_1 = 8, t_2 = 18, t_3 = 28$

So Common difference, d = 18 - 8 = 10 and first term a=8.

Now t_m = 168 (given)

We know that $T_m = a + (m - 1)d$

$\Rightarrow$ a + (m - 1)d = 168

$\Rightarrow$ 8 + (m - 1) $\times$ 10 = 168

$\Rightarrow$ (m - 1) $\times$ 10 = 160

$\implies$ m-1 = $\frac{160}{10}$

$\Rightarrow$ m - 1 = 16

Therefore, m = 17

So, t_{20} = a + (20 - 1)d

= 8 + 19 $\times$ 10

= 8 + 190

= 198

Section E

36. i. Graph of y = f(x) intersects X-axis at two distinct points. So we can say that no of zeros of y = f(x) is 2.

ii. There will not be any zero if graph of f(x) does not intersect x- axis.

iii. $x^2 + (a + 1)x + b$ is the quadratic polynomial.

2 and -3 are the zeros of the quadratic polynomial.

Thus, 2 + (-3) = $\frac{-(a+1)}{1}$

$\Rightarrow \frac{(a+1)}{1} = 1$

$\Rightarrow$ a + 1 = 1

$\Rightarrow$ a = 0

Also, 2 $\times$ (-3) = b

$\Rightarrow$ b = -6

OR

If -4 is zero of given polynomial then,

$(-4)^2 - 2(-4) - (7p + 3) = 0$

$\Rightarrow$ 16 + 8 - 7p - 3 = 0

$\Rightarrow$ 7p = 21

$\Rightarrow$ p = 3

37. i. Upper limit of modal class = 15

ii. Median class = 10 - 15

iii. a. $I = 10, f_0 = 16, f_1 = 22, f_2 = 18,\ h = 5$

$\text{Mode} = I + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h = 10 + \left(\frac{22-16}{44-16-18}\right) \times 5$

$= 13$

OR

1.

NAV (in ₹)	f	cf

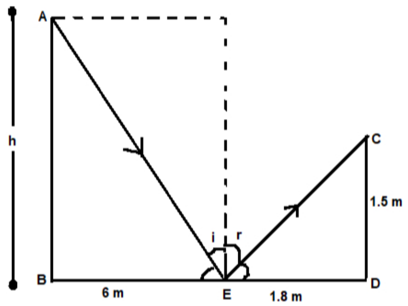
0 - 5	13	13
5 - 10	16	29
10 - 15	22	51
15 - 20	18	69
20 - 25	11	80

Median $= I + \left(\frac{\frac{N}{2}-cf}{f}\right) \times$ h $= 10 + \left(\frac{40-29}{22}\right) \times 5$

= 12.5

38. i. AA criterion

ii.



△ABE ~ △CDE (by AA criteria)

$\frac{AB}{CD} = \frac{BE}{DE}$

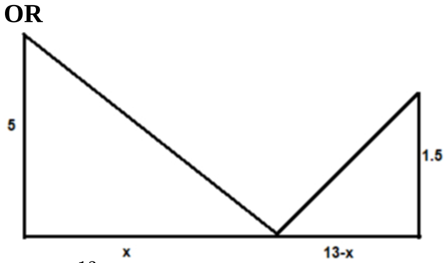
h = $\frac{6\times1.5}{1.8}$

h = 5

i.e., height of pole = 5 m.

iii. tan i = $\frac{6}{5}$

OR



$\frac{1.5}{5} = \frac{13-x}{x}$

1.5x = 65 - 5x

6.5x = 65

x = $\frac{65}{6.5}$

= 10

∴ distance of Suresh from mirror

= 13 - x

= 13 - 10

= 3 m